

MAS125 Calculus II 2006-2007

Problem sheet for exercise class 10

- There is no starred problem this week, so you can spend all your time working on the problems. The second page of questions have not yet been covered in the lectures.
- There are more problems than usual this week since they cover two week's lectures. There is not time to do all the problems in the tutorial, so it is important to study the solutions once they become available online at 3pm on Friday.
- Although tutorial problems are unassessed, they are in the same style as the problems in the final exam, so you are advised to study all of them very carefully. Note the the coursework deadline is Wednesday 4 April (although this is outside term).

Problem 1: Sketch the region of integration and then evaluate the following integrals:

$$\begin{aligned} (a) \quad & \int_0^3 \int_0^2 (4 - y^2) \, dy \, dx; & (b) \quad & \int_0^\pi \int_0^x x \sin y \, dy \, dx; \\ (c) \quad & \int_0^1 \int_0^{y^2} 3y^3 e^{xy} \, dx \, dy; & (d) \quad & \int_0^1 \int_0^{\sqrt{1-x^2}} 8y \, dy \, dx. \end{aligned}$$

Problem 2: Integrate: (a) the function $f(x, y) = x/y$ over the region bounded by the lines $y = x$, $y = 2x$, $x = 1$, $x = 2$; (b) the function $f(x, y) = x^2 + y^2$ over the triangular region with vertices $(0,0)$, $(1,0)$ and $(0,1)$.

Problem 3: Sketch the region of integration and then write an equivalent double integral with the order of integration reversed for:

$$(a) \quad \int_0^1 \int_2^{4-2x} dy \, dx; \quad (b) \quad \int_0^1 \int_y^{\sqrt{y}} dx \, dy; \quad (c) \quad \int_0^{3/2} \int_0^{9-4x^2} dy \, dx.$$

Problem 4: Sketch the region of integration, reverse the order of integration and then evaluate:

$$(a) \quad \int_0^\pi \int_x^\pi \frac{\sin y}{y} \, dy \, dx; \quad (b) \quad \int_0^1 \int_y^1 x^2 e^{xy} \, dx \, dy.$$

Problem 5: Sketch the region of integration, indicating the coordinates of the points where the boundary curves intersect, and then find the area

$$\int_0^6 \int_{y^2/3}^{2y} dx \, dy.$$

Problem 6: Find the average height of the paraboloid $z = x^2 + y^2$ over the square $0 \leq x \leq 2$, $0 \leq y \leq 2$.

Problem 7: Evaluate the following integrals by changing into polar coordinates:

$$(a) \int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) \, dx \, dy; \quad (b) \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2 + y^2 + 1) \, dx \, dy.$$

Problem 8: Find the area enclosed by one leaf of the rose $r = 12 \cos 3\theta$.

Problem 9: Evaluate the following triple integrals:

$$(a) \int_1^e \int_1^e \int_1^e \frac{1}{xyz} \, dx \, dy \, dz; \quad (b) \int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz \, dy \, dx.$$

Problem 10: Find the average value of the function $F(x, y, z) = x^2 + 9$ over the first quadrant bounded by the coordinate planes and the planes $x = 2$, $y = 2$ and $z = 2$.

Problem 11: Find the Jacobian $\partial(x, y)/\partial(u, v)$ for the transformation $x = u \cos v$, $y = u \sin v$.

Problem 12: Solve the system $u = 3x + 2y$, $v = x + 4y$ for x and y in terms of u and v . then find the Jacobian $\partial(x, y)/\partial(u, v)$. Hence find the integral

$$\int \int_R (3x^2 + 14xy + 8y^2) \, dx \, dy$$

for the region R bounded by the lines $y = -(3/2)x + 1$, $y = -(3/2)x + 3$, $y = -(1/4)x$, $y = -(1/4)x + 1$.